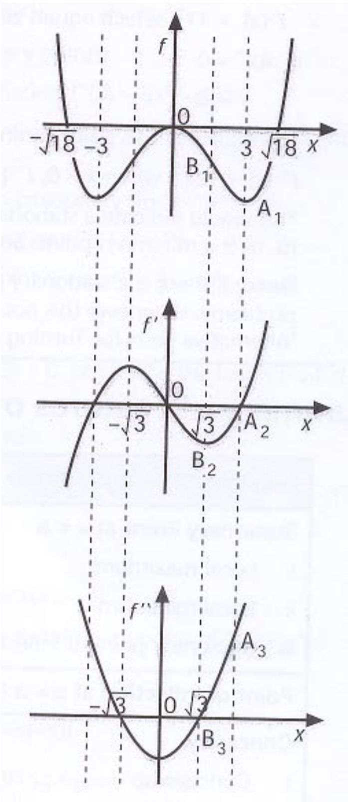
GRAPHING DERIVED FUNCTIONS

When given a graph of a function f(x) and asked to sketch its gradient function, it is best to draw the graph of f'(x) immediately below f(x) so that the scales on the x-axis line up. This allows you to note specific features on the graph of f(x) and translate it to specific features on the graph of f'(x).

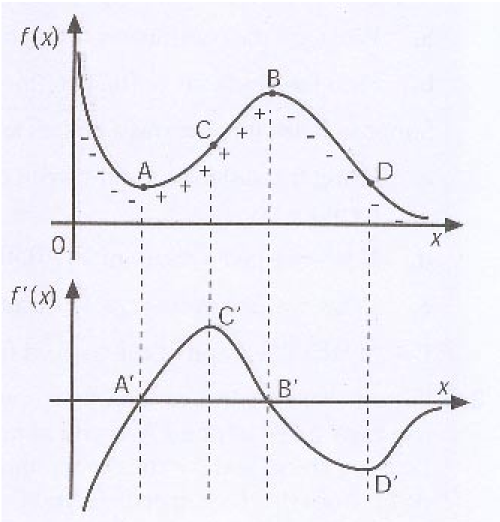
Graph 1 below is of the function f(x) = $2x^4 - 36x^2$. Note that the turning points of f(x) line up with the x-intercepts of f'(x) and the points of inflection of f(x) line up with the turning points of f'(x). In general the rules in the table below can be used to sketch gradient functions.

1)



Features of graph of $f(x)$	Features of graph of $f'(x)$
f increasing	$f'(x) > 0$
f decreasing	$f'(x) < 0$
Stationary points (maximum or minimum)	x intercepts
Points of inflection	Turning points
Stationary points of inflection	Turning points on the x-axis
Vertical asymptotes	Vertical asymptotes
Horizontal asymptotes	Horizontal asymptotes
Discontinuity or spike (abrupt change in slope)	$f'(x)$ undefined

2)

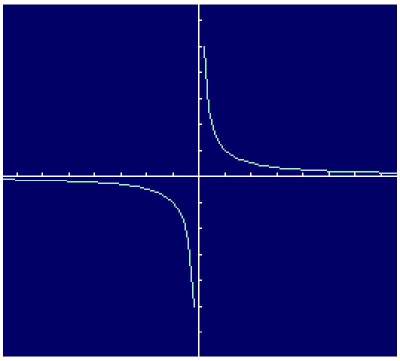


Graph 2 shows the use of the rules in the previous table to sketch the derived function:

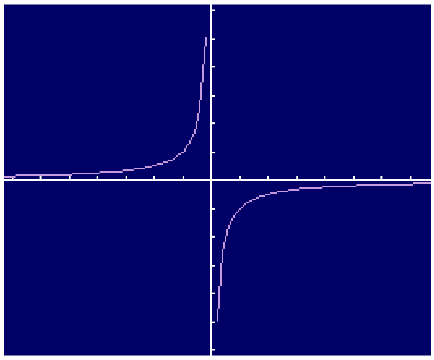
- Stationary/turning points A and B of f(x) correspond to x-intercepts A' and B' of f'(x).
- Points of inflection C and D of f(x) correspond to turning points C' and D' of f'(x).
- From 0 -> A and from B -> D, f(x) is decreasing, so f'(x) is below the x-axis.
- From A -> B, f(x) is increasing, so f'(x) is above the x-axis.

3) Also useful to note: graph of derived functions of hyperbolae

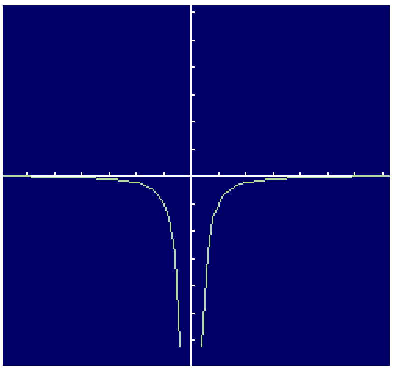
f(x) = $\frac{a}{x}$



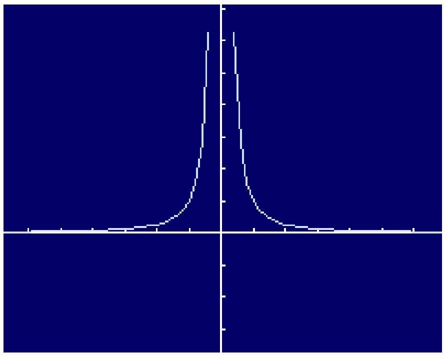
f(x) = $\frac{-a}{x}$



f'(x) = $\frac{-a}{x^2}$



f(x) = $\frac{a}{x^2}$



Exercise 3.1C M7 on CD

1 Sketch the curve $y = f(x)$ which has the following properties:

a the curve is continuous.

b $f(^-1) = f'(^-1) = f''(^-1) = 0$.

c For $x < {}^-1$ the curve is concave up.

d For $x > {}^-1$ the curve is concave down.

2 For the function $f(x) = 2x^4 + 3x^3$:

a Find the coordinates of all the stationary points and state whether they are minima, maxima or points of inflection.

b Find the non-stationary point of inflection. Justify your answer.

c Sketch the curve, clearly showing the main features.

d State the values of x for which the function is concave down.

3 For the function: $f(x) = \dfrac{x^2}{1 - x}$:

a For which value/s of x is the function not defined?

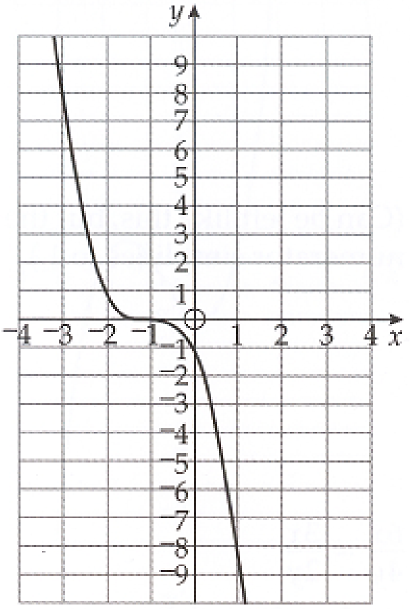
b Give the coordinates of the y-intercept.

c Give the coordinates of the x-intercepts.

d Find $f'(x)$ and hence find the coordinates of the stationary points.

Exercise 3.1C M7

1

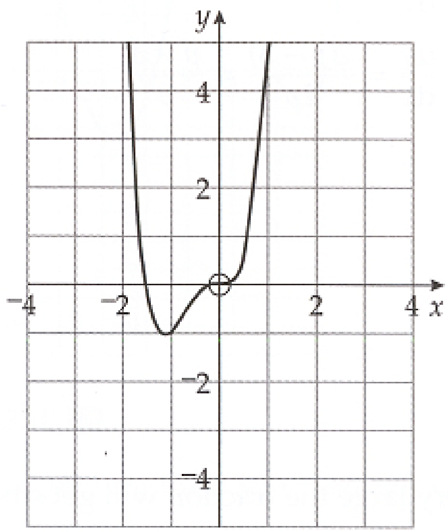


2 a When $x = 0$, $f''(x) = 0$ — point of inflection

When $x = \frac{^-9}{8}$, $f''(x) > 0$ — minimum value

b The non-stationary point is when $x = \frac{^-3}{4}$

c



Local minimum at $x = \frac{^-9}{8}$

Point of inflection at $x = 0$

x/y intercepts at $x = 0$

d Function is concave down between the two points of inflection, i.e. for $\frac{^-3}{4} < x < 0$

3 a Not defined for $x = 1$

b y-intercept at $(0, 0)$

c x-intercepts at $(0, 0)$

d $f'(x) = \frac{2x - x^2}{(1 - x)^2} = 0$

Coordinates of stationary points are $(0, 0)$ and $(2, ^-4)$